FP1 Complex Number Questions

5 (a) (i) Calculate $(2+\mathrm{i}\sqrt{5})(\sqrt{5}-\mathrm{i})$. *(3 marks)*

(ii) Hence verify that $\sqrt{5}-\mathrm{i}$ is a root of the equation

$$(2+\mathrm{i}\sqrt{5})z = 3z^*$$

where z^* is the conjugate of z. *(2 marks)*

(b) The quadratic equation

$$x^2 + px + q = 0$$

in which the coefficients p and q are real, has a complex root $\sqrt{5}-\mathrm{i}$.

(i) Write down the other root of the equation. *(1 mark)*

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6 It is given that $z = x + \mathrm{i}y$, where x and y are real numbers.

(a) Write down, in terms of x and y, an expression for

$$(z+\mathrm{i})^*$$

where $(z+\mathrm{i})^*$ denotes the complex conjugate of $(z+\mathrm{i})$. *(2 marks)*

(b) Solve the equation

$$(z+\mathrm{i})^* = 2\mathrm{i}z + 1$$

giving your answer in the form $a+b\mathrm{i}$. *(5 marks)*

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1 (a) Solve the following equations, giving each root in the form $a+b\mathrm{i}$:

(i) $x^2 + 16 = 0$; *(2 marks)*

(ii) $x^2 - 2x + 17 = 0$. *(2 marks)*

(b) (i) Expand $(1+x)^3$. *(2 marks)*

(ii) Express $(1+\mathrm{i})^3$ in the form $a+b\mathrm{i}$. *(2 marks)*

(iii) Hence, or otherwise, verify that $x = 1+\mathrm{i}$ satisfies the equation

$$x^3 + 2x - 4\mathrm{i} = 0$$ *(2 marks)*

3 It is given that $z = x + \mathrm{i}y$, where x and y are real numbers.

(a) Find, in terms of x and y, the real and imaginary parts of

$$z - 3\mathrm{i}z^*$$

where z^* is the complex conjugate of z. *(3 marks)*

(b) Find the complex number z such that

$$z - 3\mathrm{i}z^* = 16$$ *(3 marks)*

FP1 Complex Number Answers

5(a)(i)	Full expansion of product	M1		
	Use of $i^2 = -1$	m1		
	$(2+\sqrt{5}i)(\sqrt{5}-i) = 3\sqrt{5}+3i$	A1	3	$\sqrt{5}\sqrt{5} = 5$ must be used – Accept not fully simplified
(ii)	$z^* = x - iy\ (=\sqrt{5}+i)$	M1		
	Hence result	A1	2	Convincingly shown (AG)
(b)(i)	Other root is $\sqrt{5}+i$	B1	1	

6(a)	$(z+i)^* = x - iy - i$	B2	2	
(b)	$\ldots = 2ix - 2y + 1$	M1		$i^2 = -1$ used at some stage
	Equating R and I parts	M1		involving at least 5 terms in all
	$x = -2y + 1,\ -y - 1 = 2x$	A1√		ft one sign error in (a)
	$z = -1 + i$	m1A1√	5	ditto; allow $x = -1,\ y = 1$
	Total		**7**	

1(a)(i)	Roots are $\pm 4i$	M1A1	2	M1 for one correct root or two correct factors
(ii)	Roots are $1 \pm 4i$	M1A1	2	M1 for correct method
(b)(i)	$(1+x)^3 = 1 + 3x + 3x^2 + x^3$	M1A1	2	M1A0 if one small error
(ii)	$(1+i)^3 = 1 + 3i - 3 - i = -2 + 2i$	M1A1	2	M1 if $i^2 = -1$ used
(iii)	$(1+i)^3 + 2(1+i) - 4i$	M1		with attempt to evaluate
	$\ldots = (-2+2i) + (2-2i) = 0$	A1	2	convincingly shown (**AG**)
	Total		**10**	

3(a)	Use of $z^* = x - iy$	M1		
	$z - 3iz^* = x + iy - 3ix - 3y$	m1		Condone sign error here
	$R = x - 3y,\ I = -3x + y$	A1	3	Condone inclusion of i in I Allow if correct in (b)
(b)	$x - 3y = 16,\ -3x + y = 0$	M1		
	Elimination of x or y	m1		
	$z = -2 - 6i$	A1F	3	Accept $x = -2,\ y = -6$; ft $x + 3y$ for $x - 3y$
	Total		**6**	
